

Vertical Ascent to Geosynchronous Orbit with Constrained Thrust Angle

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A discussion is presented about the overall payload ratio of a solid-rocket-booster-assisted laser propulsion system for geosynchronous-Earth-orbit payload launch with thrust-angle constraint. A trajectory shaping method is used for constructing the vertical ascent trajectory in the equatorial plane. There is a thrust angle under which the payload ratio (ratio of final mass to initial mass) is to be maximized. This maximum payload can be obtained only when the ground station can provide the required peak laser power. Now, if the peak laser power is limited to a lower level, the payload ratio that can be obtained will also be lower. When the peak laser power that can be provided is too low, the thrust-to-weight ratio will be less than one at the initial phase of the launch. To keep the performance of the launching system, we include in the model a strap-on solid rocket booster to compensate the system for thrust loss. It is found that the penalty can be reduced significantly.

Nomenclature

A_N	= normal component of aerodynamic force, N
A_T	= tangential component of aerodynamic force, N
$A(t)$	= dV/dt , m/s^2
a	= sound speed, m/s
a_0, a_1, a_2, \dots	= coefficients of $A(t)$ polynomial
C_D	= drag coefficient
D	= drag, N
\bar{D}	= D/m
g	= Earth's gravitational acceleration, m/s^2
g_R	= gravitational acceleration at the Earth's surface, m/s^2
h_0	= characteristic altitude
I_{sp}	= specific impulse, s
I_{spl}	= specific impulse of laser propulsion system, s
I_{sps}	= specific impulse of solid rocket booster, S
(\hat{i}_c, \hat{i}_r)	= unit vectors of circumferential-radial coordinate system
(\hat{i}_t, \hat{i}_n)	= unit vectors of tangential-normal coordinate system
J	= performance index
l	= effective path length through the atmosphere
M_r	= V_r/a , relative Mach number
m	= mass of vehicle, kg
\bar{m}	= m/m_i
P	= laser power, W
R	= radius of the Earth, m
r	= radial position of vehicle, m
S	= reference area of vehicle, m^2
T	= thrust, N
\bar{T}	= T/m_i
T_l	= thrust of laser propulsion system, N
\bar{T}_l	= T_l/m_i
T_s	= thrust of solid rocket booster, N
\bar{T}_s	= T_s/m_i
t	= times, s

t_s	= operation time of solid rocket booster, s
V	= speed of vehicle, m/s
V_r	= relative speed of vehicle with respect to the atmosphere, m/s
α	= thrust angle
β	= inverse of atmospheric scale height, $1/m$
γ	= flight-path angle
ε	= atmospheric absorption coefficient
η	= efficiency of the laser system
η_a	= atmospheric absorption efficiency
η_d	= atmospheric diffraction efficiency
η_h	= thrust efficiency
θ	= longitude
μ	= gravitational constant of the Earth, m^3/s^3
ρ	= atmospheric density, kg/m^3
ω	= Earth's rotation rate

Subscripts

f	= final condition
i	= initial condition

Introduction

IN Ref. 1 the idea of vertical ascent to geosynchronous Earth orbit (GEO) was introduced. By vertical ascent the vehicle is launched and propelled straight up and always stays directly above the launching site. Hence, if the ascending trajectory is contained in the equatorial plane, then the vehicle will stay at the GEO when the GEO's altitude is reached with no residual radial velocity component. The main advantage of vertical ascent is that the advanced propulsion systems, such as laser propulsion, can be used.²

In Refs. 3 and 4 investigations have been given to the optimal trajectory for vertical ascent to GEO. Two constraints were considered: the thrust acceleration constraint is $2.5g_R$, and the dynamic pressure constraint is $30,000 \text{ N/m}^2$. From the result the conclusion was made that the vertical ascent is a very promising method of launching the GEO satellite. More than 10% of the payload ratio (ratio of final mass to initial mass) was obtained under the assumption that enough laser power can be provided from the ground station and that the specific impulse $I_{sp} = 1500 \text{ s}$ can be achieved. For the currently used chemical launch vehicle, the GEO payload capability is no more than 1%. In Ref. 4 we further analyzed the performance requirements of a laser propulsion system. The following affecting factors and efficiency parameters have been considered: atmospheric (expressed

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as transmission efficiency), laser-beam spreading (expressed as diffraction efficiency), and thruster efficiency. The peak laser power required for $I_{sp} = 1500$ s and $m_i = 10,000$ kg is 4.5 GW.

In Ref. 5 the effect of laser-power limitation on the payload ratio has been studied. The available peak laser power was reduced from 4.5 to 4 GW, then to 3.5 GW, and so forth. The purpose of this paper is to investigate the effect of the thrust-angle constraint on the payload ratio. We shall also consider a combined laser and chemical propulsion system when there is a reduction in the available peak laser power.⁶ A strap-on solid rocket booster (SRB) will be used.

Problem Formulation

Equations of Motion

Consider a launch vehicle treated as a point-mass model. The spherical atmosphere is rotating along with the Earth. For flight in the equatorial plane, the two-dimensional equations of motion are⁶

$$\frac{dr}{dt} = V \sin \gamma \quad (1a)$$

$$\frac{d\theta}{dt} = \frac{V \cos \gamma}{r} \quad (1b)$$

$$\frac{dV}{dt} = \frac{T \cos \alpha + A_T}{m} - g \sin \gamma + \omega^2 r \sin \gamma \quad (1c)$$

$$\frac{d\gamma}{dt} = \frac{T \sin \alpha + A_N}{mV} - \frac{g \cos \gamma}{V} + \frac{V \cos \gamma}{r} + 2\omega + \frac{\omega^2 r \cos \gamma}{V} \quad (1d)$$

$$\frac{dm}{dt} = -\frac{T}{g_R I_{sp}} \quad (1e)$$

The calculation of the aerodynamic force is based on the relative speed between the vehicle and the rotating atmosphere:

$$V = V \sin \gamma \quad (2)$$

The gravitational acceleration g can be expressed as $g = \mu/r^2$, where the value of μ for the Earth is given in Table 1 (Ref. 7). The geometry of flight within the equatorial plane is shown in Fig. 1. When the vehicle is launched and thrusts straight up and always stays directly above the launching site, the particular trajectory is called the vertical ascent. From Eq. (1b) we have the relation³

$$\frac{d\theta}{dt} = \frac{V \cos \gamma}{\gamma} = \omega \quad \text{or} \quad \theta = \omega t \quad (3)$$

where we have assumed that $\theta = 0$. Consequently, the five equations of motion in Eqs. (1) are reduced to three:

$$\frac{dV}{dt} = \frac{T \cos \alpha + A_T}{m} - g \sin \gamma + \omega^2 r \sin \gamma \quad (4a)$$

$$\frac{d\gamma}{dt} = \frac{T \sin \alpha + A_N}{mV} - \frac{g \cos \gamma}{V} + \frac{V \cos \gamma}{r} + 2\omega + \frac{\omega^2 r \cos \gamma}{V} \quad (4b)$$

$$\frac{dm}{dt} = -\frac{T}{g_R I_{sp}} \quad (4c)$$

Table 1 Physical parameters of the Earth and vehicle

Parameter	Value
Radius of the Earth R	6.378152×10^6 m
Angular rate of the Earth's rotation ω	7.2921151×10^{-5} rad/s
Gravitational constant μ	3.9859898×10^{14} m ³ /s ²
Gravitational acceleration at the Earth's surface g_R	9.7982035 m/s ²
Circumferential speed at the Earth's surface	4.6510219×10^2 m/s
Radius of GEO r_f	4.2164121×10^7 m
Circumferential speed at GEO V_f	3.0746562×10^3 m/s
Initial mass of vehicle m_i	10,000 kg (Ref. 7)
Reference area of vehicle S	81.5 m ² (Ref. 7)
Specific impulse of vehicle I_{sp}	1500 s

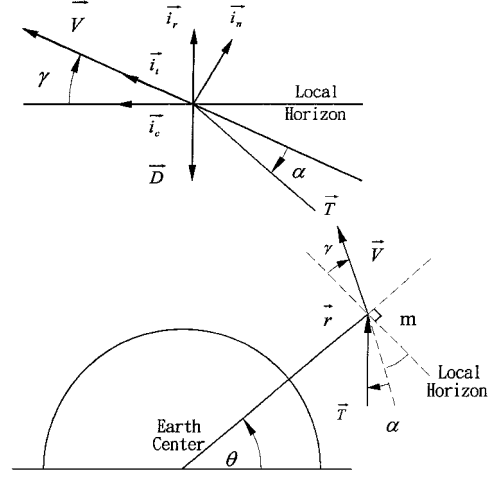


Fig. 1 Geometry of flight in the equatorial plane.

where r is obtained from the relation

$$r = V \cos \gamma / \omega \quad (5)$$

Furthermore, the assumption is made that there is no lift control and the only aerodynamic force considered is the drag. Referring to Fig. 1, we have

$$A_T \mathbf{i}_t + A_N \mathbf{i}_n = \mathbf{D} = -D \sin \gamma \mathbf{i}_t - D \cos \gamma \mathbf{i}_n \quad (6)$$

The drag coefficient C_D is a function of the relative Mach number⁸ defined as

$$M_r = V_r / a \quad (7)$$

Both ρ and a are functions of the radial distance r . In differential form the atmospheric density can be expressed as

$$\frac{d\rho}{\rho} = -\beta(r) dr \quad (8)$$

where $\beta(r)$ is the inverse of the atmospheric scale height.

By defining the dimensionless mass \bar{m} and the modified thrust \bar{T} and drag \bar{D} as

$$\bar{m} = m / m_i, \quad \bar{T} = T / m_i, \quad \bar{D} = D / m_i \quad (9)$$

Eqs. (4) become the modified form

$$\frac{dV}{dt} = \frac{\bar{T} \cos \alpha - \bar{D} \sin \gamma}{\bar{m}} - g \sin \gamma + \omega^2 r \sin \gamma \quad (10a)$$

$$\frac{d\gamma}{dt} = \frac{\bar{T} \sin \alpha - \bar{D} \cos \gamma}{\bar{m} V} - \frac{g \cos \gamma}{V} + \frac{V \cos \gamma}{r} + 2\omega + \frac{\omega^2 r \cos \gamma}{V} \quad (10b)$$

$$\frac{d\bar{m}}{dt} = -\frac{\bar{T}}{g_R I_{sp}} \quad (10c)$$

The dimensionless mass has the initial value $\bar{m}_i = 1$, and its final value is the fraction of the initial mass that remains when the vehicle arrives at the GEO. The acceleration caused by the thrust is

$$(\text{thrust acceleration}) = T / m = \bar{T} / \bar{m} \quad (11)$$

A relation between the thrust magnitude and the thrust angle can be derived by taking the time derivative of Eq. (5), and using the differential relations in Eqs. (1a) and (10a):

$$\bar{T} = \frac{4\omega \bar{m} V \sin \gamma}{\cos(\alpha + \gamma)} \quad (12)$$

Parameterization and Trajectory Shaping

The performance index is the maximization of the final mass with a given initial mass. It is equivalent to the maximization of the payload or the minimization of the propellant consumption. We have

$$J = \max \bar{m}_f \quad (13)$$

To solve a two-point boundary-value problem, there was a technique in which the control was parameterized as a polynomial function of the normalized time.⁹ In Ref. 4 we used the technique differently. Instead of the control variable we parameterized the acceleration of the vehicle, that is,

$$\frac{dV}{dt} = A(t) = a_0 + a_1 t + a_2 t^2 + \dots \quad (14)$$

The integration of Eq. (14) gives

$$V(t) - V(0) = a_0 t + \frac{1}{2} a_1 t^2 + \frac{1}{3} a_2 t^3 + \dots \quad (15)$$

Therefore, it is also a technique of trajectory shaping with the vehicle speed shaped as a polynomial function of the flight time. In

terms of the parameters a_0, a_1, a_2, \dots , the control variable \bar{T} can be expressed as³

$$\bar{T} = (1/\cos \alpha) \{ \bar{D} \sin \gamma + \bar{m} [A(t) + g \sin \gamma - \omega V \sin \gamma \cos \gamma] \} \quad (16)$$

The thrust angle can be calculated from the relation

$$\tan \alpha = \frac{\cos \gamma}{\sin \gamma} - \frac{4\omega \bar{m} V}{\bar{D} \sin \gamma + \bar{m} [A(t) + g \sin \gamma - \omega V \sin \gamma \cos \gamma]} \quad (17)$$

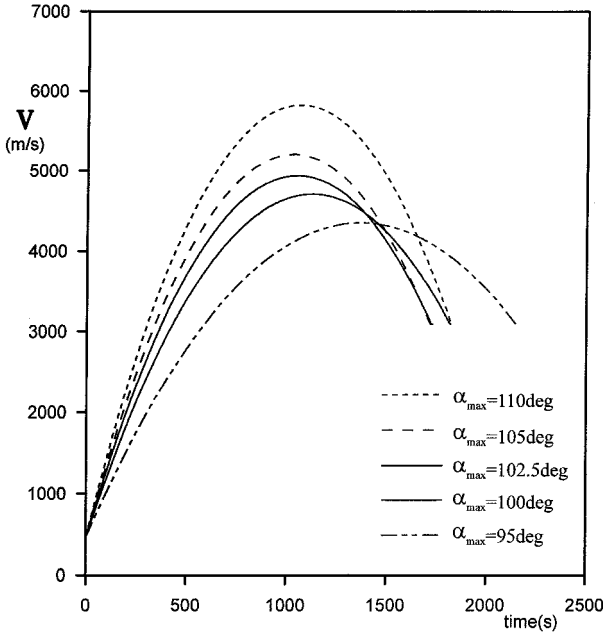
Constraints

Two constraints have been considered previously: the maximum thrust acceleration is constrained at $2.5g_R$, and the maximum dynamic pressure is constrained at $30,000 \text{ N/m}^2$ (Refs. 1–3). To satisfy the two constraints, we have the following:

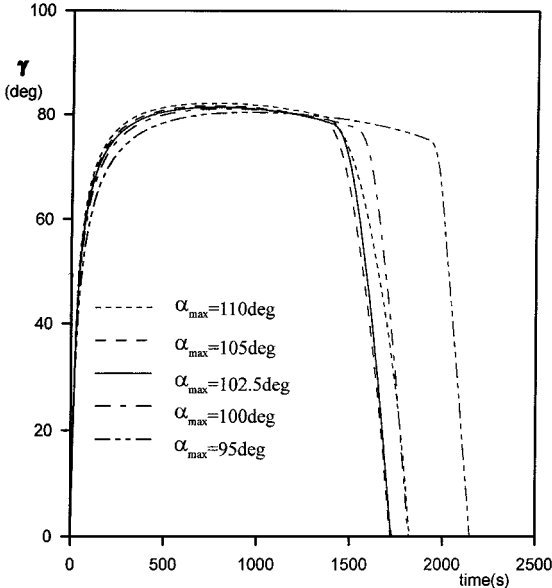
1) If $T/m = \bar{T}/\bar{m} \geq 2.5g_R$, then

$$\bar{T} = 2.5g_R \bar{m} \quad (18a)$$

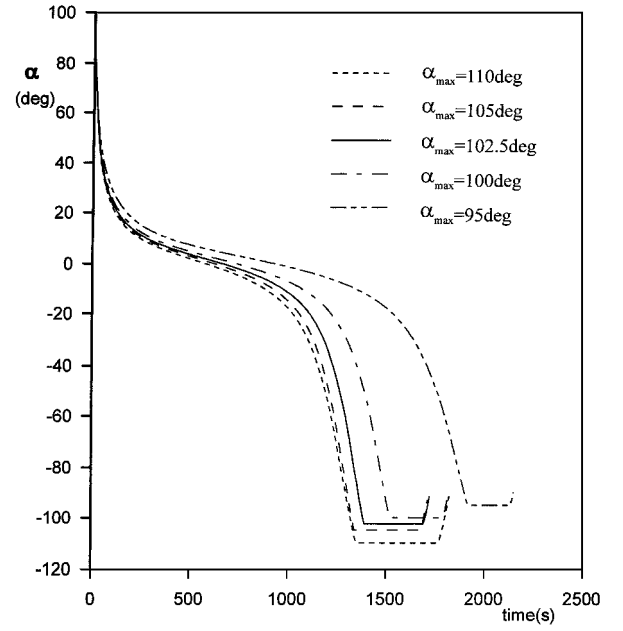
$$\cos(\alpha + \gamma) = \frac{4\omega V \sin \gamma}{2.5g_R} \quad (18b)$$



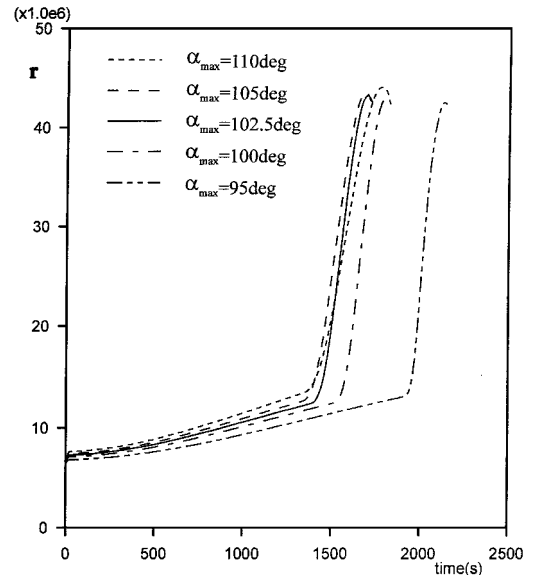
a) Vehicle speed



b) Flight-path angle

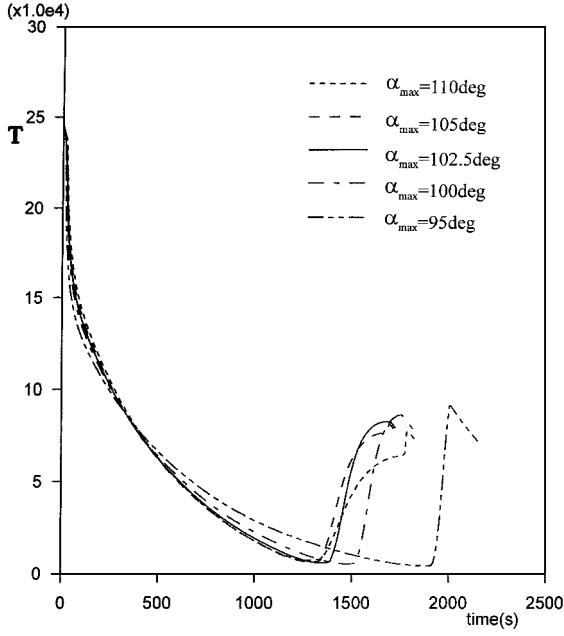


c) Thrust angle

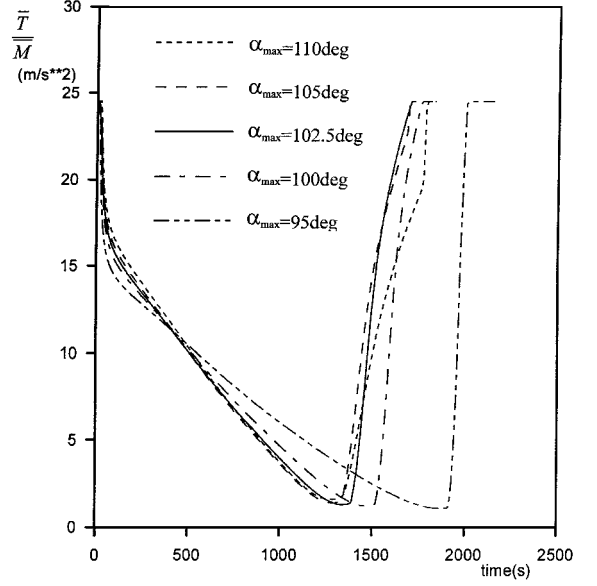


d) Vehicle position

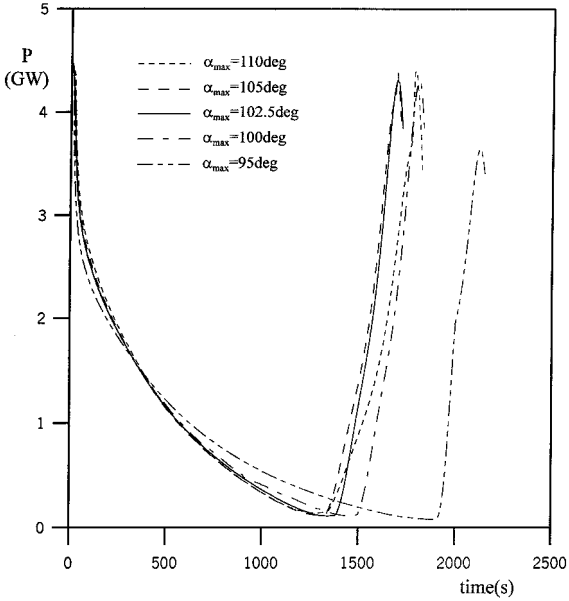
Fig. 2 State and control variable for $\alpha_{\max} = 110, 105, 102.5, 100$, and 95 deg, respectively.



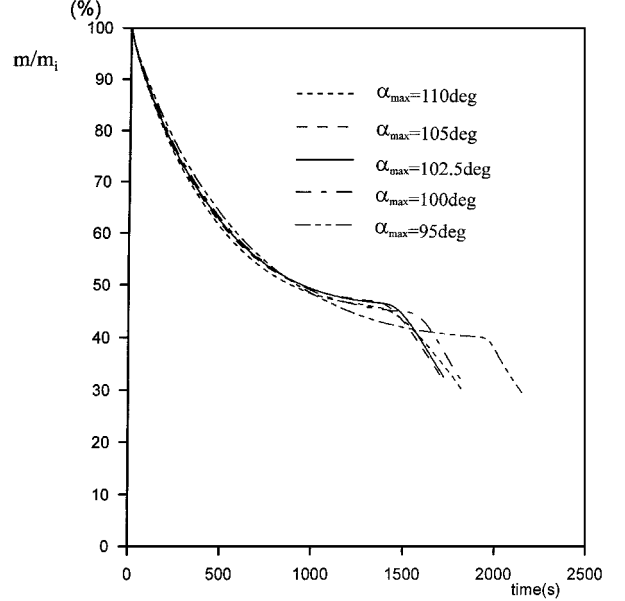
e) Thrust magnitude



g) Thrust acceleration



f) Laser power



h) Vehicle mass

Fig. 2 State and control variable for $\alpha_{\max} = 110, 105, 102.5, 100$, and 95 deg, respectively (continued).

2) If $\frac{1}{2}\rho V_r^2 \geq 30,000$, then

$$\bar{T} = \frac{980\omega\bar{m}}{\sqrt{\rho}\cos(\alpha + \gamma)} \quad (19a)$$

$$\sin(\alpha + \gamma) = (1/\bar{T}) \left[\bar{D} + \bar{m}g + \frac{1}{2}\beta(r)\bar{m}V^2\sin^2\gamma - 4\omega\bar{m}V\cos\gamma \right] \quad (19b)$$

In this paper one more constraint will be imposed—the thrust angle constraint, that is,

3) If

$$|\alpha| \leq \alpha_{\max}, \quad \text{then} \quad |\alpha| = \alpha_{\max} \quad (20)$$

Numerical Computation and Results

Efficiency of a Laser Propulsion System

Let the total efficiency of the laser propulsion system be denoted by η . Its relation with the thrust, specific impulse, and laser power can be expressed as¹⁰

$$\bar{T} = \frac{2\eta P}{m_i I_{sp} g_R} \quad (21)$$

Usually the total efficiency can be decomposed into three efficiencies: the thruster efficiency η_h , the atmospheric absorption efficiency η_a , and the atmospheric diffraction efficiency η_d . In mathematical form¹¹

$$\eta = \eta_h \eta_a \eta_d \quad (22)$$

For vertical ascent the total efficiency of a laser propulsion system can be expressed as^{5,6}

$$\eta = 0.4e^{-\varepsilon l} \left[1 - 4(h/h_0)^2 \right] \quad (23)$$

where $\varepsilon = 1.25 \times 10^{-5} \text{ m}^{-1}$ and $h_0 = 10^8 \text{ m}$.

Vertical Ascent Trajectories

For the problem of ground-to-orbit launch that we considered, the laser propulsion system uses a large ground-fixed laser generator to

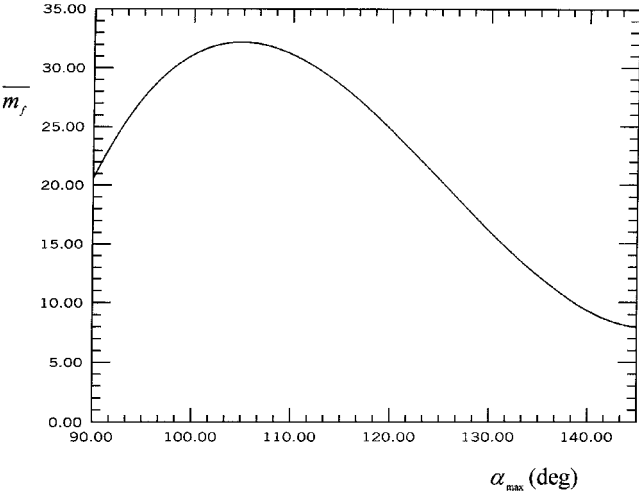


Fig. 3 Variation of maximum \bar{m}_f with α_{\max} .

supply energy to heat an inert propellant in a rocket thruster on the launch vehicle. The two potential advantages of such a system are the simplicity of the thruster and the possibility of very high specific impulse. The trajectory without thrust-angle constraint had been solved in Ref. 4 by using $I_{sp} = 1500$ s and a linear function of dV/dt . Other physical parameters of the Earth and vehicle are listed in Table 1. For numerical computation the initial and final conditions to be used are

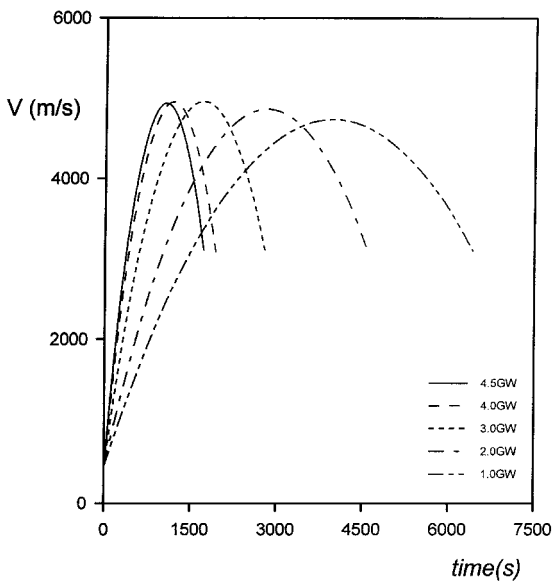
$$(t_i, V_i, \gamma_i, \bar{m}_i) = (0, 465.17304 \text{ m/s}, 1 \text{ deg}, 1) \quad (24a)$$

$$(t_f, V_f, \gamma_f, \bar{m}_f) = (\text{free}, 3074.6562 \text{ m/s}, 0 \text{ deg}, \max) \quad (24b)$$

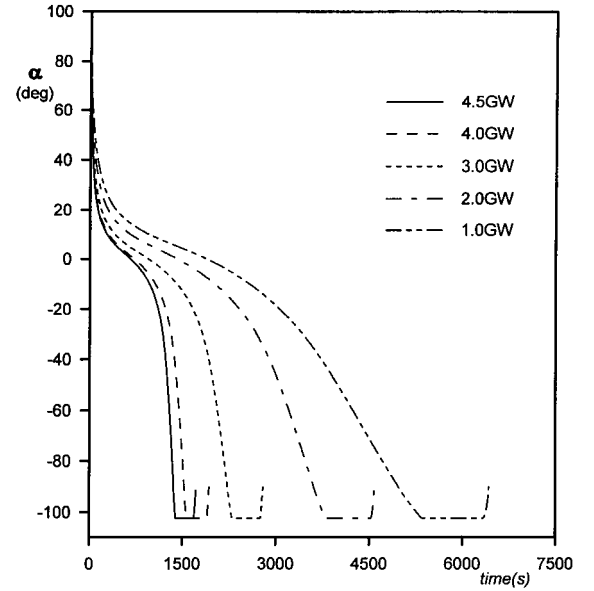
When two terms in Eq. (15) are considered, we have

$$a_0 t_f - \frac{1}{2} a_1 t_f^2 = 2609.4832 \quad (25)$$

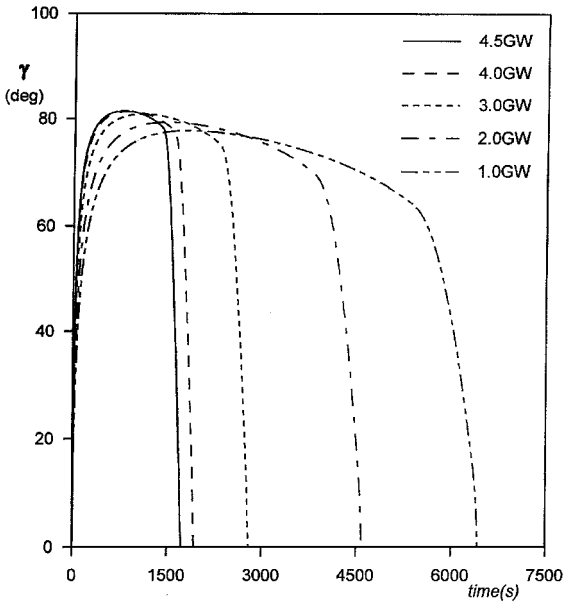
The trajectories with different values of α_{\max} have been solved and presented in Fig. 2. The parameters a_0 , a_1 , and t_f are listed in Table 2. Figure 3 shows that we have the global maximum \bar{m}_f when α_{\max} is specified at 102.5 deg.



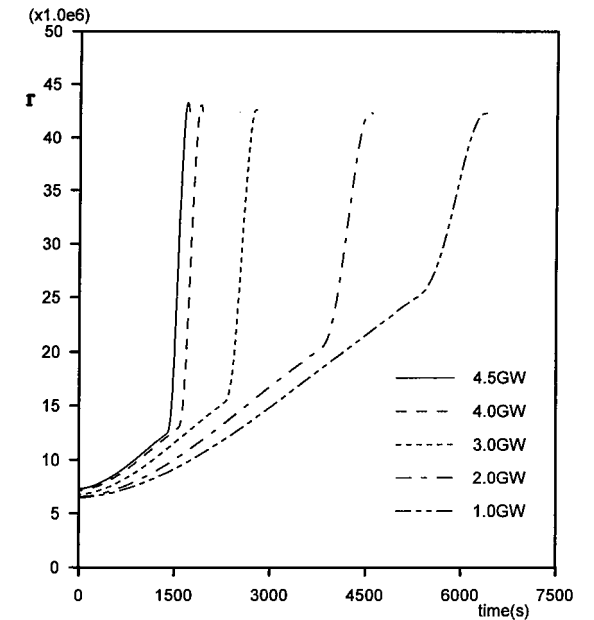
a) Vehicle speed



c) Thrust angle

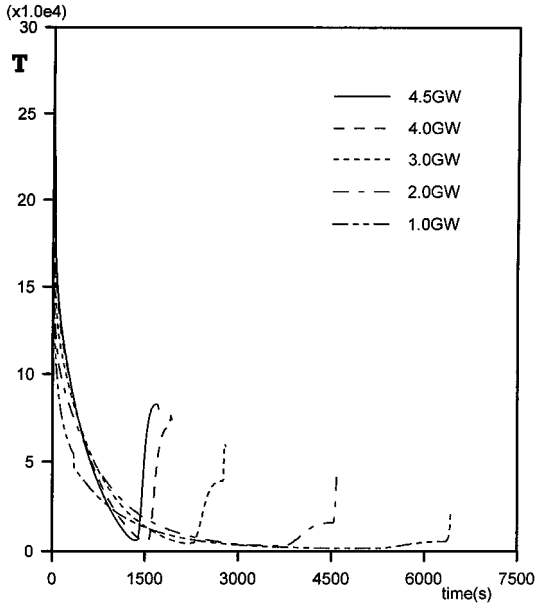


b) Flight-path angle

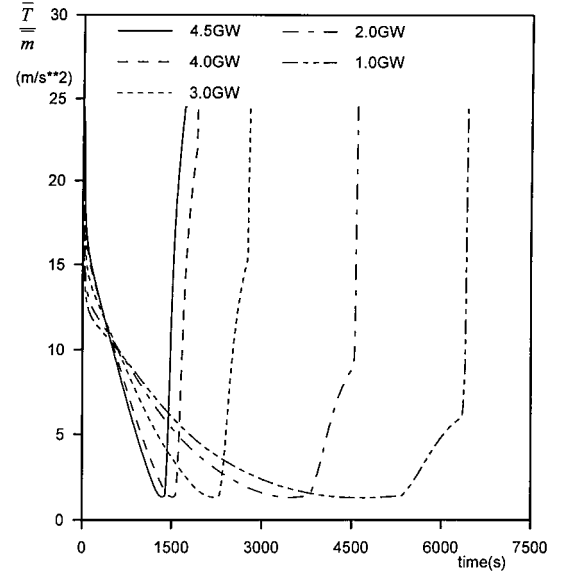


d) Vehicle position

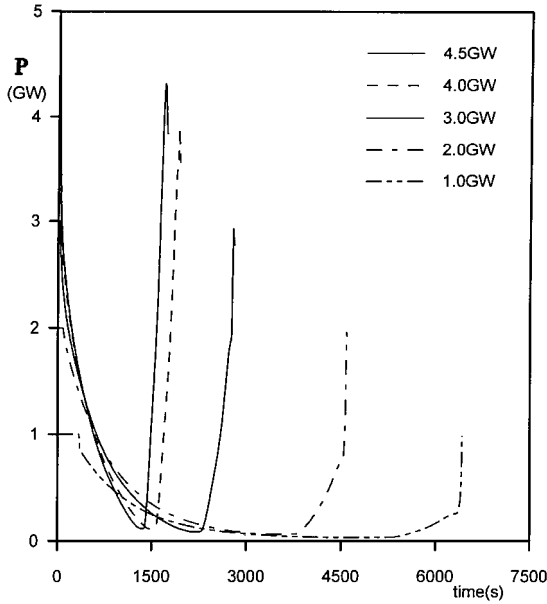
Fig. 4 State and control variables for $\alpha_{\max} = 102.5$ deg and $P_{\max} = 4.5, 4.0, 3.0, 2.0$, and 1.0 GW, respectively.



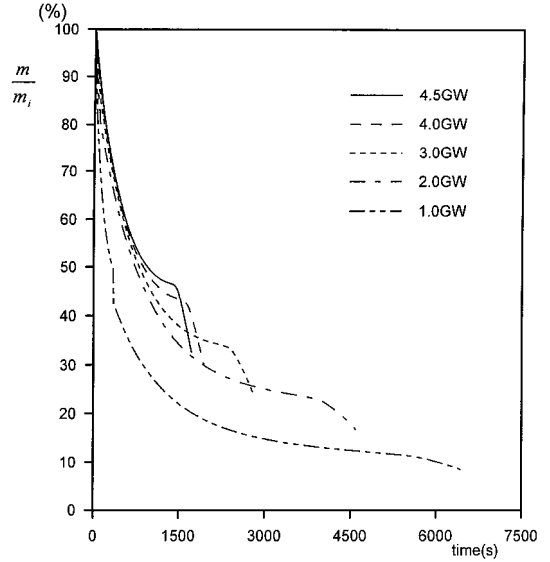
e) Thrust magnitude



g) Thrust acceleration



f) Laser power



h) Vehicle mass

Fig. 4 State and control variables for $\alpha_{\max} = 102.5$ deg and $P_{\max} = 4.5, 4.0, 3.0, 2.0$, and 1.0 GW, respectively (continued).

SRB-Assisted Laser Propulsion System

SRB will be used to compensate the launching system for thrust loss caused by maximum available laser-power limitation. The specific impulse of the SRB is assumed to be 250 s. In this way the total thrust will be the sum of the thrust generated by the laser propulsion system and the thrust from the SRB:

$$T = T_1 + T_s \quad (26)$$

or

$$\bar{T} = \bar{T}_1 + \bar{T}_s \quad (27)$$

Equation (10c) will be changed to the form

$$\frac{d\bar{m}}{dt} = -\frac{\bar{T}_1}{g_R I_{sp1}} - \frac{\bar{T}_s}{g_R I_{sp2}} \quad (28)$$

In Eq. (26) the total thrust is the required thrust control for the trajectory shaping, and the laser propulsion thrust is dependent on the available laser power. Thus we have

$$\bar{T}_s = \bar{T}_1 - \frac{2\eta P}{m_1 I_{sp1} g_R} \quad (29)$$

When the available maximum laser power is 4.5 GW, the total thrust required can be provided by the laser propulsion system, and no SRB is needed. Now, if the available maximum laser power is limited to 4.0 GW, an SRB must be strapped on for a certain time interval at the starting phase of the flight. As shown in Table 3, the laser power required is greater than 4.0 GW before $t = 18.1$ s. Therefore, the operation time of the SRB will be from $t = 0$ to 18.1 s. The total mass of the SRB is 174.73 kg, which consists of 148.52 kg (85%) of propellant and 26.21 kg (15%) of structural mass. The structural mass will be jettisoned when the SRB burns out. The final mass obtained is 3010 kg and is 30.10% of the initial mass. This overall payload ratio is 2.22% lower than the maximum value of 32.32%. The α_{\max} constraint is 102.5 deg.

Secondly, we consider the two cases with the limitation of maximum laser power at 3.5 and 3.0 GW, respectively. In the case of the 3.5 GW limitation, we need an SRB to operate 21.4 s. The SRB has a total mass of 363.3 kg, which consists of 308.8 kg of propellant and 54.5 kg of structure. The overall payload ratio is 27.12%. In the case of the 3.0 GW limitation, the required SRB has an operation time of 26.3 s and a total mass of 554.98 kg. The overall payload ratio obtained is 23.93%, and the flight time is increased to 2793.1 s.

Table 2 Thrust angle, a_0 , a_1 , flight time, and payload

α , deg	$a_0/9.8$	a_1	t_f	m_f/m_i , %
± 125	1.30	-1.0198×10^{-2}	2323.1	19.58
± 120	1.17	-9.4114×10^{-3}	2182.0	23.99
± 115	1.09	-9.3639×10^{-3}	2002.8	27.01
± 110	1.03	-9.1111×10^{-3}	1820.8	29.88
± 105	0.94	-8.9588×10^{-3}	1716.8	31.94
± 102.5	0.87	-8.1360×10^{-3}	1723.2	32.32
± 100	0.77	-6.7238×10^{-3}	1816.9	31.80
± 95	0.58	-4.3019×10^{-3}	2113.6	29.63

Table 3 Laser power, SRB mass, a_0 , a_1 , flight time, and payload ratio

P_{\max} , GW	SRB mass, KG	t_s , s	$a_0/9$	a_1	t_1 , s	m_f/m_i , %
4.5	0	0	0.87	-8.1368×10^{-3}	1723.2	32.32
4.0	174.73	18.1	0.78	-6.5171×10^{-3}	1930.6	30.10
3.5	363.30	21.4	0.66	-4.6550×10^{-3}	2288.5	27.12
3.0	554.98	26.3	0.54	-3.1198×10^{-3}	2793.1	23.93
2.5	792.34	41.6	0.42	-1.9012×10^{-3}	3557.5	20.36
2.0	1262.45	89.2	0.32	-1.1192×10^{-3}	4586.3	16.57
1.5	2381.78	195.0	0.25	-6.9545×10^{-4}	5736.3	12.48
1.0	4463.45	341.2	0.22	-5.4464×10^{-4}	6424.1	8.36

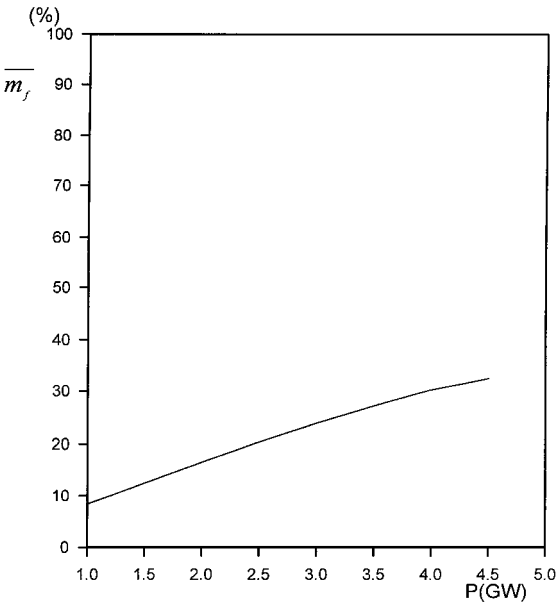


Fig. 5 Overall payload ratio as a function of the laser power limitation for $\alpha_{\max} = 102.5$ deg.

Further numerical computation has been done for maximum laser power limited to 2.5, 2.0, 1.5, and 1.0 GW, respectively. The trajectories are shown in Fig. 4. Finally, the overall payload ratio as a function of the laser-power limitation for $\alpha_{\max} = 102.5$ deg is shown in Fig. 5.

Conclusions

Under the constraints of thrust angle, thrust acceleration, and dynamic pressure, and the consideration of the laser propulsion system efficiency, expressed as the product of the thrust efficiency, the atmospheric absorption efficiency, and the atmospheric diffraction efficiency, the thrust history for vertical ascent to GEO trajectory shaping has been solved with a given specific impulse value. There is a particular thrust-angle constraint at which the overall payload ratio (the ratio of final mass to initial mass) has a global maximum value. The maximum laser power is required at the initial time. When the available maximum laser power is reduced, the performance index is also reduced. A strap-on SRB is used to compensate the launching system for the thrust loss caused by the laser-power limitation. Finally, the overall payload ratio as a function of the laser-power limitation is presented, with the thrust-angle constraint fixed at the best value.

References

¹Martin, J. A., "Vertical Ascent to Geosynchronous Orbit," *Journal of Guidance, Control, and Dynamics*, Vol. 7, No. 6, 1984, pp. 757, 758.

²Kare, J. T., "Pulsed Laser Propulsion for Low Cost, High Volume Launch to Orbit," International Astronautical Federation Conf. on Space Power, UCRL-101139, Cleveland, OH, Oct. 1989.

³Chern, J. S., Hong, Z. C., and Cheng, W. L., "Optimal Vertical Ascent to GEO with Thrust Acceleration and Dynamic Pressure Constraints," *Acta Astronautica*, Vol. 35, No. 1, 1995, pp. 9-18.

⁴Hong, Z. C., Chang, C. Y., and Chern, J. S., "Laser Propulsion System Performance Requirements for Single-Stage to GEO Launch," International Astronautical Federation, IAF-94-S.3.421, Jerusalem, Oct. 1994; also *Space Technology*, Vol. 15, No. 6, 1995, pp. 385-401.

⁵Chern, J. S., Hong, Z. C., and Liu, J. M., "Effect of Laser Power Limitation on Single-Stage to GEO Launch," American Astronomical Society, AAS-95-449, Halifax, NS, Canada, Aug. 1995.

⁶Hong, Z. C., and Liu, J. M., and Chern, J. S., "Overall Payload Ratio of a Combined Laser and Chemical Propulsion System for GEO Launch," International Astronautical Federation, IAF-95-S.4.03, Oslo, Oct. 1995.

⁷Williamson, W. E., "Minimum and Maximum Endurance Trajectories for Gliding Flight in a Horizontal Plane," *Journal of Guidance and Control*, Vol. 2, No. 6, 1979, pp. 157-163.

⁸Schottle, U. M., Grallert, H., and Hewitt, F. A., "Advanced Air-Breathing Propulsion Concepts for Winged Launch Vehicles," International Astronautical Federation, IAF-88-248, Bangalore, India, Oct. 1988.

⁹Glumb, R. J., and Krier, H., "Concepts and Status of Laser-Supported Rocket Propulsion," *Journal of Spacecraft and Rockets*, Vol. 21, No. 1, 1984, pp. 70-79.

¹⁰Kare, J. T., "Laser-Supported Detonation Waves and Pulsed Laser Propulsion," 17th International Shockwave and Shock Tube Symposium, Bethlehem, PA, July 1989.

¹¹Kare, J. T., "Trajectory Simulation for Laser Launching," *Proceedings of the 1986 SDIO/DARPA Workshop on Laser Propulsion*, Vol. 2, 1986, pp. 61-77.

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